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# A criterion for leading-edge separation

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The maximum angle of attack for unseparated flow over an airfoil of chord $c$ with finite nose radius of curvature $r$ is shown to be $0.818(r/c)^{\frac{1}{2}}$.

## 1. Introduction

This paper addresses a part of the problem of aerodynamic stall, namely the determination of the critical angle of attack, above which the laminar boundary layer separates near the leading edge.

Any wing will stall at sufficiently large angle of attack. Put the other way, any reasonably well-designed wing will *not* stall, and hence will generate lift that increases very nearly linearly with angle of attack, for sufficiently small angle of attack. Such an increase will continue until the angle of attack reaches the critical value at which stall does occur, namely that in which the point of boundary-layer separation on the upper surface moves rapidly forward. The lift reaches a maximum near this critical angle of attack, then falls rapidly.

Much of the wing design methodology has always been about postponing stall to as high an angle of attack as possible. The literature is extensive, and will not be reviewed here. It is clear that the 'rounder' the nose, the higher the permissible angle of attack; sharp-edged wings stall very quickly. So everyone knows that the maximum angle of attack (and hence the maximum lift) increases with thickness, or more accurately with nose radius of curvature. But at what rate?

The purpose of the present note is to show theoretically that, at least as a prediction of laminar leading-edge separation, the appropriate rate of increase is as the square root of the nose radius, and specifically that the angle of attack in radians at which leading-edge separation first occurs is proportional to the square root of the ratio of nose radius to chord, the coefficient of proportionality being about 0.818. An equivalent proportionality was described as 'well-known' by Lighthill (1951, p. 209).

Leading-edge separation usually accompanies stall, but is neither necessary (in principle) nor sufficient for its occurrence. For most normal airfoils, it can indeed be considered *necessary*; catastrophic loss of lift is unlikely to occur until the separation point is close to the leading edge. However, for most airfoils, leading-edge separation is not quite *sufficient* for stall, since reattachment of the separated boundary layer can occur, with continued lift generation in a flow with a thin 'bubble' on its upper surface, up to higher angles of attack. Hence the present theory tends to underestimate the maximum useful angle of attack.

It is appropriate first to give a quick summary of the way in which this theory works. Some of the required extra detail is presented in the following sections, or can be extracted from the uniformly valid approximations derived by Lighthill (1951).

According to thin-airfoil theory, the fluid velocity near the leading edge of an airfoil at angle of attack $\alpha$ to a stream $U$ behaves like the inverse square root of distance from that edge. If the airfoil has zero thickness, this infinite velocity is inevitable within inviscid fluid theory, and a real fluid would separate immediately from such a sharp leading edge.

On the other hand, if the airfoil has non-zero thickness, with a rounded leading edge of radius of curvature $r$ satisfying $r \ll c$, where $c$ is the chord of the airfoil, then the apparent inverse-square-root leading-edge singularity simply models high but not infinite velocities at which the flow passes around the leading edge. Locally, the airfoil can be replaced by the parabola which touches its nose. The exact potential flow around that parabola can be written down in closed form as a combination of the uniform stream $U$ and a 'turning' flow whose velocity components vary as the inverse square root of distance from the focus of the parabola, and are thus finite on its surface, but can be matched in its far field with the apparent singularity of the thin-airfoil solution.

By carrying out this matching explicitly, this local flow can be shown to be characterized by a single non-dimensional parameter

$$\beta = \alpha(2c/r)^{\frac{1}{2}}. \tag{1.1}$$

If the airfoil has camber, it is assumed that $\alpha$ is measured relative to the 'ideal' angle of attack, so that there is no leading-edge singularity when $\alpha = 0$. Then at $\beta = 0$ the local flow is symmetric, with its stagnation point at the vertex of the parabola, and a decreasing pressure everywhere on the parabola's surface. On the other hand, for $\beta > 0$ the stagnation point moves to the lower surface, and there is a maximum-velocity point on the upper surface, followed by a region of increasing pressure.

We now ask the question whether the laminar boundary layer on the parabola does or does not separate in the deceleration region downstream of the maximum-velocity point. Certainly it does not separate in the symmetric flow at $\beta = 0$ which has no such adverse-pressure zone, and also not for small $\beta$, when the deceleration is gentle, but separation is inevitable for sufficiently large $\beta$. The critical value of $\beta$ is 1.157; that is, there is no separation for $\beta < 1.157$ and separation for $\beta > 1.157$, or

$$\alpha > 0.818(r/c)^{\frac{1}{2}}. \tag{1.2}$$

The actual laminar boundary-layer computation leading to this result is an old one (Werle & Davis 1972). However, the identification (1.1) of the parameter $\beta$ with a scaled angle of attack has not been made before, and we now derive this result using ideas of matched expansions (Van Dyke 1964).

## 2. Outer expansion

We assume the usual conditions for thin-airfoil theory in an 'outer' domain whose fundamental lengthscale is the chord $c$. The airfoil is supposed to have top and bottom surfaces

$$y = -\alpha x + f_C(x) \pm f_T(x), \quad 0 \leqslant x \leqslant c, \tag{2.1}$$

where the angle of attack $\alpha$, the camber $f_C(x)$, and the thickness $2f_T$ are all small quantities. The camber function $f_C(x)$ is assumed to be bounded together with its derivatives, but need not vanish at the ends. The thickness function $f_T(x)$ must vanish at both ends, and its slope must be bounded at the trailing end $x = c$. However, at the leading end $x = 0$, the thickness behaves like the square root of $x$, specifically

$$f_T(x) = (2rx)^{\frac{1}{2}} + O(x^{\frac{3}{2}}), \quad x \downarrow 0. \tag{2.2}$$

Now if the fluid velocity is $\nabla(Ux+\phi)$, where $\phi$ is the disturbance velocity potential due to the airfoil, then the linearized boundary condition on the airfoil is

$$v = \phi_x = U[-\alpha + f'_{\mathrm{C}}(x) \pm f'_{\mathrm{T}}(x)], \quad y = 0_{\pm}. \tag{2.3}$$

The solution of Laplace's equation subject to (2.3) and suitable conditions at infinity, on the wake, and the Kutta condition at the trailing edge proceeds in a standard way, and the result is that on the airfoil the streamwise disturbance velocity is (Newman 1977, p. 164 *et seq.*)

$$u = \phi_x = \pm U\alpha\left(\frac{c-x}{x}\right)^{\frac{1}{2}} \pm \frac{U}{\pi}\left(\frac{c-x}{x}\right)^{\frac{1}{2}} \int_0^c \frac{f'_{\mathrm{C}}(\xi)}{x-\xi}\left(\frac{\xi}{c-\xi}\right)^{\frac{1}{2}} \mathrm{d}\xi + \frac{U}{\pi}\int_0^c \frac{f'_{\mathrm{T}}(\xi)}{x-\xi}\mathrm{d}\xi. \tag{2.4}$$

The corresponding lift coefficient per unit span is

$$C_L = 2\pi\alpha - \frac{4}{c}\int_0^c f'_{\mathrm{C}}(x)\left(\frac{x}{c-x}\right)^{\frac{1}{2}} \mathrm{d}x. \tag{2.5}$$

We are particularly interested in the behaviour of this thin-airfoil solution near the leading edge $x = 0$, namely from (2.4) to leading order

$$u \to \pm U\left(\frac{c}{x}\right)^{\frac{1}{2}}\left[\alpha - \frac{1}{\pi}\int_0^c \frac{f'_{\mathrm{C}}(\xi)}{[\xi(c-\xi)]^{\frac{1}{2}}}\mathrm{d}\xi\right]. \tag{2.6}$$

We now assume without loss of generality that the integral in (2.6) is zero, i.e. that the orientation at zero angle of attack is taken as that 'ideal' value (Abbott & von Doenhoff 1958, p. 70) for which there is no leading-edge singularity. If the airfoil is cambered, this means that it has non-zero lift at zero angle of attack. Thus (2.6) reduces simply to

$$u \to \pm U\alpha\left(\frac{c}{x}\right)^{\frac{1}{2}}. \tag{2.7}$$

Note that so long as $f_{\mathrm{T}}(x)$ satisfies (2.2), the thickness contribution to the $x$-wise leading-edge velocity is bounded for small $x$, and therefore negligible compared to the inverse-square-root behaviour in (2.7). On the other hand, the thickness distribution dominates the $y$-wise velocity, namely from (2.3) and (2.2)

$$v \to \pm U\left(\frac{r}{2x}\right)^{\frac{1}{2}}. \tag{2.8}$$

We now turn to the inner expansion, valid in an 'inner' domain near the leading edge, of lengthscale $r$, where the velocities will match those given by (2.7) and (2.8). Before doing that, however, we should note that in such a domain the expected orders of magnitude of $u$ and $v$ will be the same as each other, since the fluid particles there move in more-or-less circular arcs clockwise around the nose. If this is so, then by comparing (2.7) and (2.8) we must have $\alpha = O(r/c)^{\frac{1}{2}}$ or $\beta = O(1)$, where $\beta$ is the parameter defined by (1.1).

## 3. Inner expansion

It is appropriate to write down at once a non-dimensional one-parameter complex velocity potential that solves the inner problem, and to state its properties, namely

$$F(Z) = Z + (\beta - \mathrm{i})(2Z-1)^{\frac{1}{2}} \tag{3.1}$$

with the branch cut to the right along the positive real axis from $Z = 0.5$. In the complex $Z = X + \mathrm{i}Y$ plane, the curve

$$(2Z-1)^{\frac{1}{2}} = Y + \mathrm{i} \tag{3.2}$$

defines the parabola $Y^2 = 2X$ with unit nose radius of curvature and focus at $Z = 0.5$. Combining (3.1) and (3.2) shows that the stream function, the imaginary part of $F(Z)$, takes the constant value $\beta$ on this parabola, which therefore can be taken as an impermeable boundary surface.

The flow represented by (3.1) consists of a unit $X$-directed uniform stream, combined with a flow whose velocity components vary as the inverse square root of distance from the focus point $Z = 0.5$ inside the parabola. The complex velocity is

$$F'(Z) = 1 + (\beta - \mathrm{i})(2Z-1)^{-\frac{1}{2}} \tag{3.3}$$

everywhere, and in particular on the parabola's surface

$$F'(Z) = (Y+\beta)/(Y+\mathrm{i}) \tag{3.4}$$

with magnitude $$Q = (Y+\beta)/(Y^2+1)^{\frac{1}{2}}.$$

Clearly $Q = 0$ at the stagnation point $Y = -\beta$. This result agrees with Lighthill's (1951) equation (45).

As $|Z| \to \infty$ on the parabola,

$$F'(Z) \to 1 + (\beta - \mathrm{i})/Y. \tag{3.5}$$

Hence the $X$-wise disturbance velocity tends to $\pm\beta(2X)^{-\frac{1}{2}}$, while the $Y$-wise disturbance velocity tends to $\pm(2X)^{-\frac{1}{2}}$.

Given all the above properties of the non-dimensional potential $F(Z)$, we need merely observe that in the present problem, the complex potential $UF(x/r + \mathrm{i}y/r)$ satisfies all requirements for the inner expansion. That is, it generates as a streamline the parabola $y^2 = 2rx$ which is the inner limiting form of the airfoil geometry at its nose, and its disturbance velocity components tend to those given by (2.7) and (2.8) as $x \to \infty$.

The whole inner problem is thus parametrized by $\beta$, and only by $\beta$. When $\beta$ is zero, or $\alpha = 0$, the inner flow is symmetric with stagnation at the nose and a positive pressure coefficient that decays monotonically to zero downstream. Boundary-layer separation is not possible on a symmetric parabola. As soon as we let $\beta > 0$, the stagnation point moves to the lower surface, and there is a maximum-velocity point on the upper surface. Thus, on the upper surface the pressure coefficient reaches a negative minimum value (which can easily be seen to be exactly $-\beta^2$) at $Y = 1/\beta$, before increasing toward zero far downstream. When $\beta$ is large, i.e. for large angle of attack, or small nose radius, or sharp leading edges, this pressure minimum is large and narrow, and boundary-layer separation is inevitable shortly downstream of it.

This inner flow was discussed by Van Dyke (1956) and the laminar boundary layer on the upper surface of the parabola was computed by Werle & Davis (1972). This boundary-layer computation was re-checked as part of the present study, and the critical value $\beta = 1.157$ for separation appears to be accurate.

## 4. Conclusion

The very simple law suggested here is not a stall prediction, but rather a leading-edge separation prediction. It therefore underpredicts the angle of attack for maximum lift, as in the NACA airfoil data tabulated in Abbott & von Doenhoff

(1958), by factors of about 0.5 to 0.8. However, it may have a role to play in qualitative airfoil design.

The present result depends on the leading edge being smooth and rounded. It fails where there are sharp-edged leading-edge devices such as slots. Of course this is as it should be, since these devices are used precisely in order to do better than rounded leading edges can ever do in delaying separation. It should be possible to adapt the present inner and outer expansion procedure (cf. Moriarty & Tuck 1989) to enable the effect of leading-edge devices to be predicted.

Another challenging extension would be to compute flows in the angle of attack range between that where leading-edge separation first occurs and that where true stall occurs and the lift reaches its maximum value. Such flows would have a thin circulating separation bubble attached to the upper surface, reminiscent of attached cavitation bubbles (Tulin & Hsu 1980) or constant-vorticity 'Batchelor' flows (Hurley 1989, p. 397).

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